Why Hydrodynamics?

or why not?

Hydrodynamics

Bathtub

Hydrodynamics

Bathtub

Bigger Bathtub

Hydrodynamics that looks like it should not work

Hydraulic jump

Hydrodynamics that looks like it should not work

Foil Surfing

Navier

Navier-Stokes Equation ~1820

$$\boxed{\rho\left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v}\right] = -\nabla P + \eta\nabla^2\mathbf{v}}$$

Density

Velocity

Pressure

Viscosity

Stokes

Navier

Stokes

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CMI

Millennium Problem #4 = 1,000,000$

Navier-Stokes Equation: Do solutions always exist, and are they unique?

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Density Velocity Pressure Viscosity

Stokes

CMI

Millennium Problem #4 = 1,000,000$

Navier-Stokes Equation: Do solutions always exist, and are they unique?

Olga Ladyzhenskaya (~1960): In 2 dimensions, yes.

… but still unproven in 3 dimensions.

“Hydrodynamics” applies to (among many many other things):

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Vehicle drag

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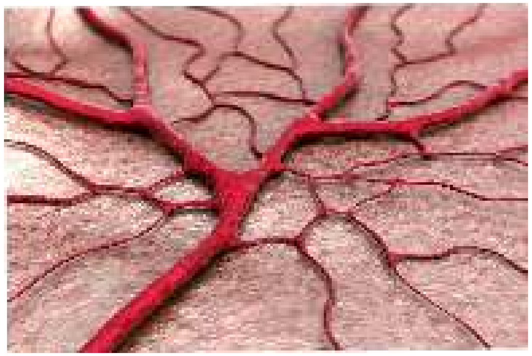
Vehicle drag



Blood Flow

“Hydrodynamics” applies to (among many many other things):

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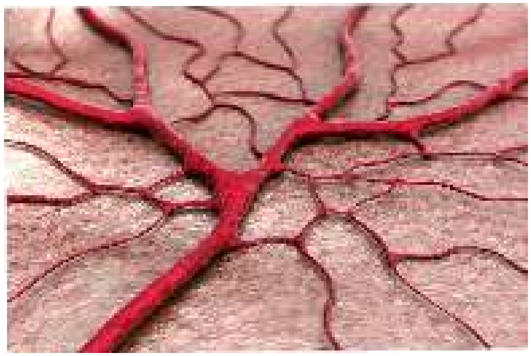

Blood Flow

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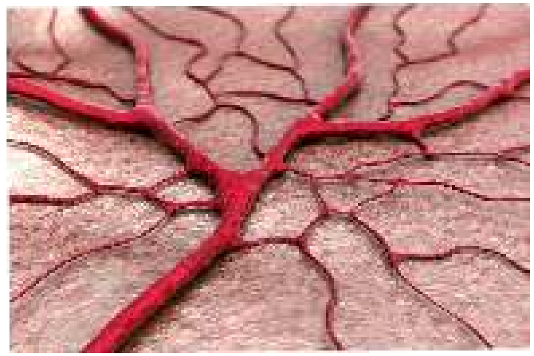

Blood Flow

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8,000 kg sailboat sky-leap

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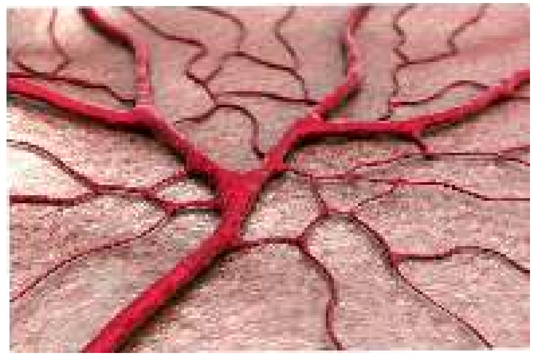

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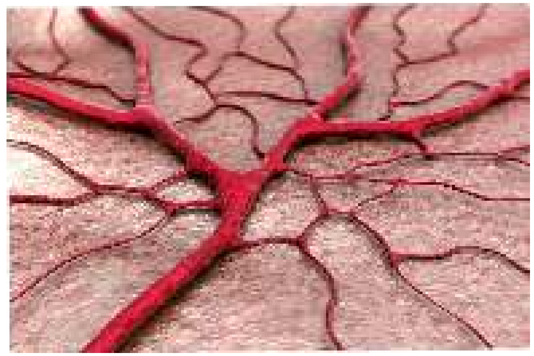


Blood Flow

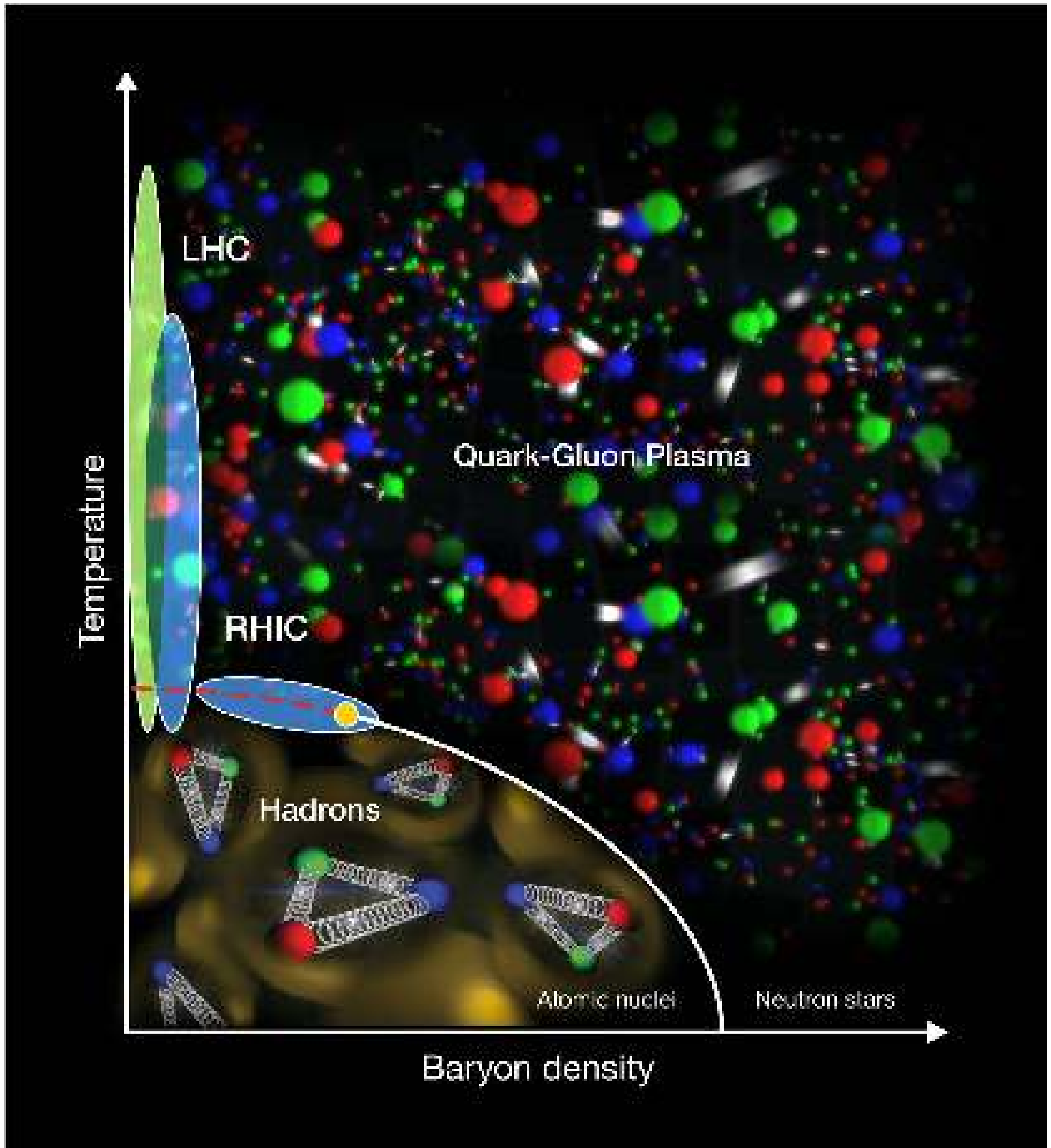
Aircraft

Clouds (Helmholtz instability)

8,000 kg sailboat sky-leap

Hurricane

Hydrodynamics of quark-gluon plasma: length ~10^{-14} m energy ~10^{13} K

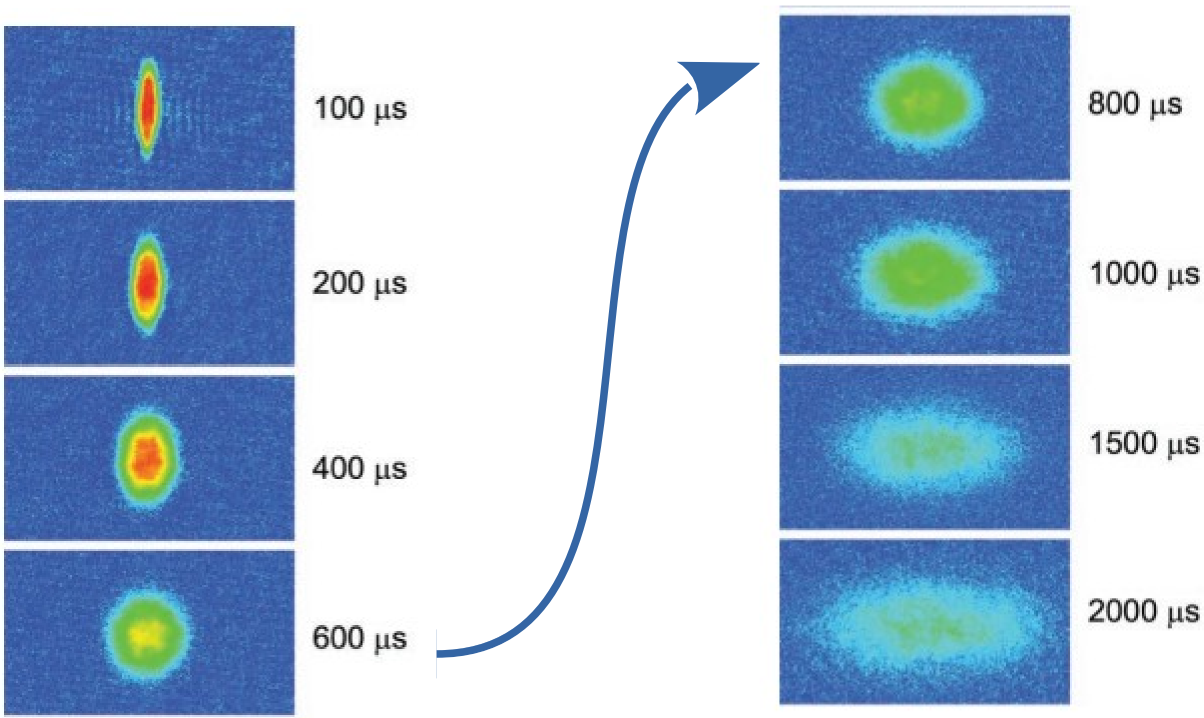


Turbulent Hydrodynamics of Interstellar Medium (Gases, Ions, etc..)

Length scale .. up to 1000 light years ~ 10^{19} m

Carina Nebula (photo from Hubble)

Cold Trapped Atoms: energy ~ 10^{-7} K



Energy scales: 10^{-7} K - 10^{13} K

Length scales: 10^{-14} m - 10^{19} m

Energy scales: 10^{-7} K - 10^{13} K

Length scales: 10^{-14} m - 10^{19} m

Why is Hydrodynamics so ubiquitous?

Why is Hydrodynamics so ubiquitous?

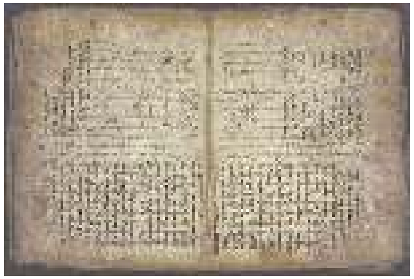
Where does Hydrodynamics come from?

Why is Hydrodynamics so ubiquitous?

Where does Hydrodynamics come from?

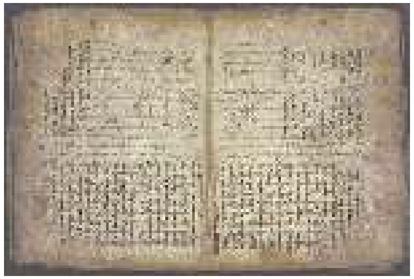
Who does Hydrodynamics come from?

Archimedes (ca 250 BCE)



“On floating bodies”

Archimedes (ca 250 BCE)



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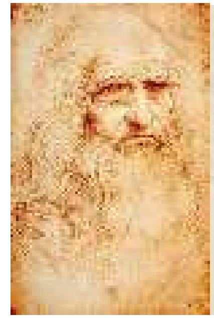

da Vinci (~1510)

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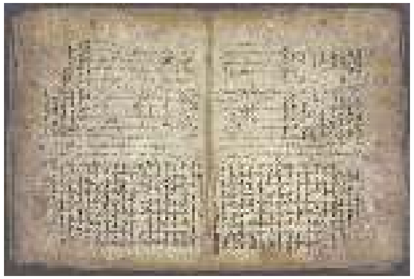

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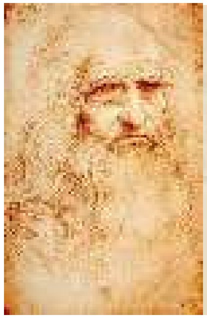

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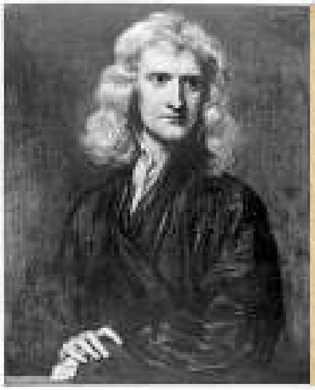

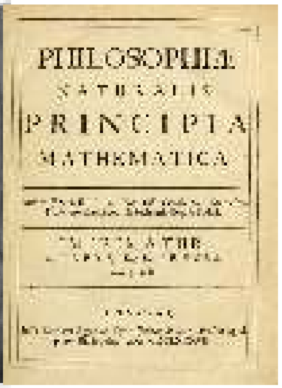

Isaac Newton (1686)

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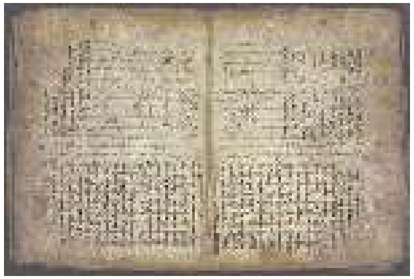

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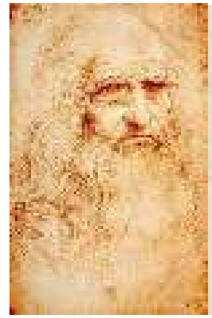

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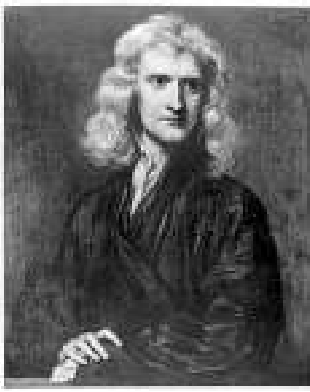

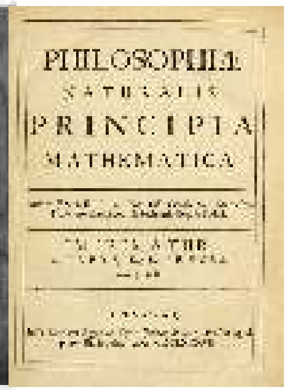

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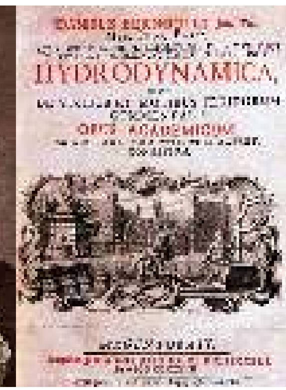
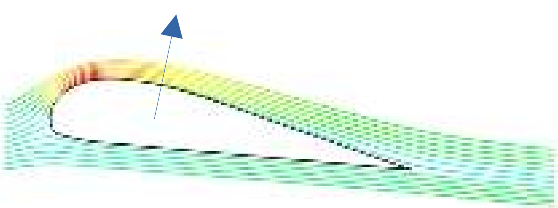


Daniel Bernoulli (1738)



Leonhard Euler (1757)

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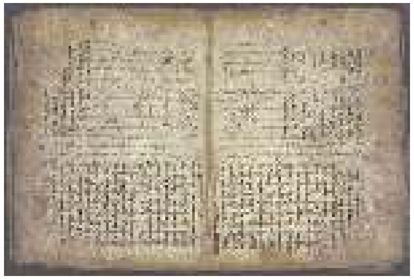

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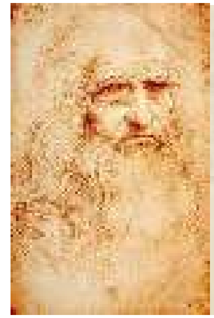

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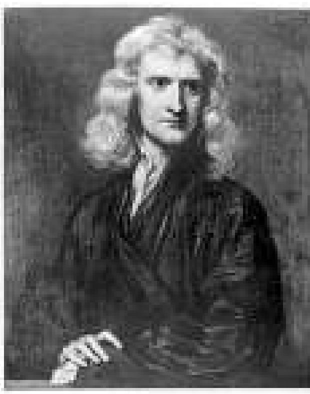
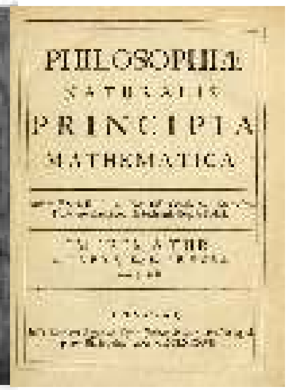
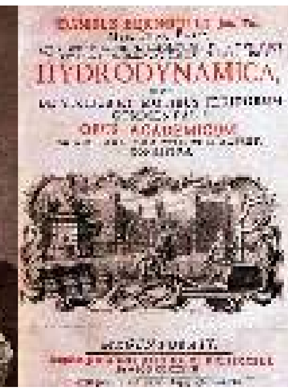




PHILOSOPHIÆ NATURALIS PRINCIPIA MATHEMATICA

Isaac Newton (1686)



HYDRODYNAMICA,

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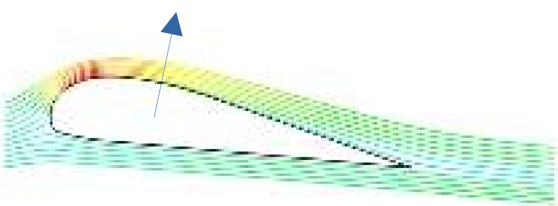

PRINCIPES GÉNÉRAUX

Leonhard Euler (1757)

… and...
Navier-Stokes
~1820

All of that was long before we knew that fluids are made of microscopic particles

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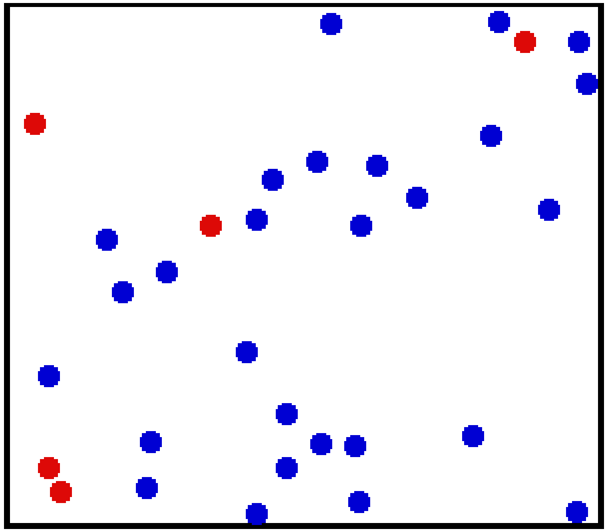
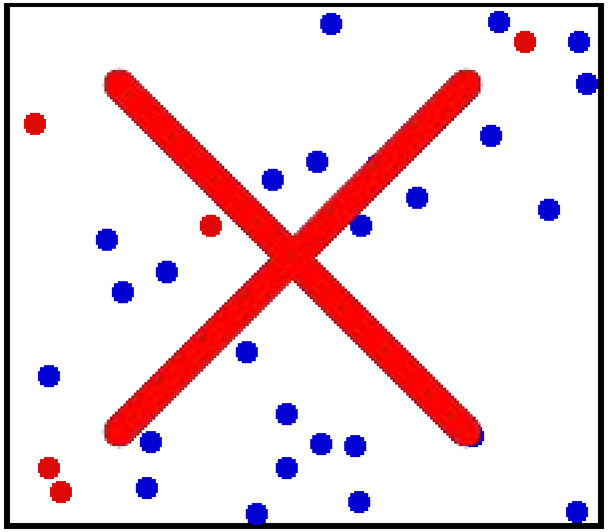


Kinetic theory of gas, Boltzmann-Maxwell … mid 1800’s

All of that was long before we knew that fluids are made of microscopic particles



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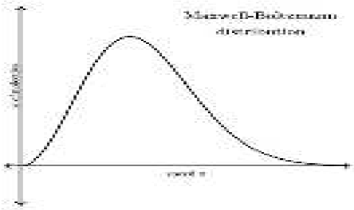
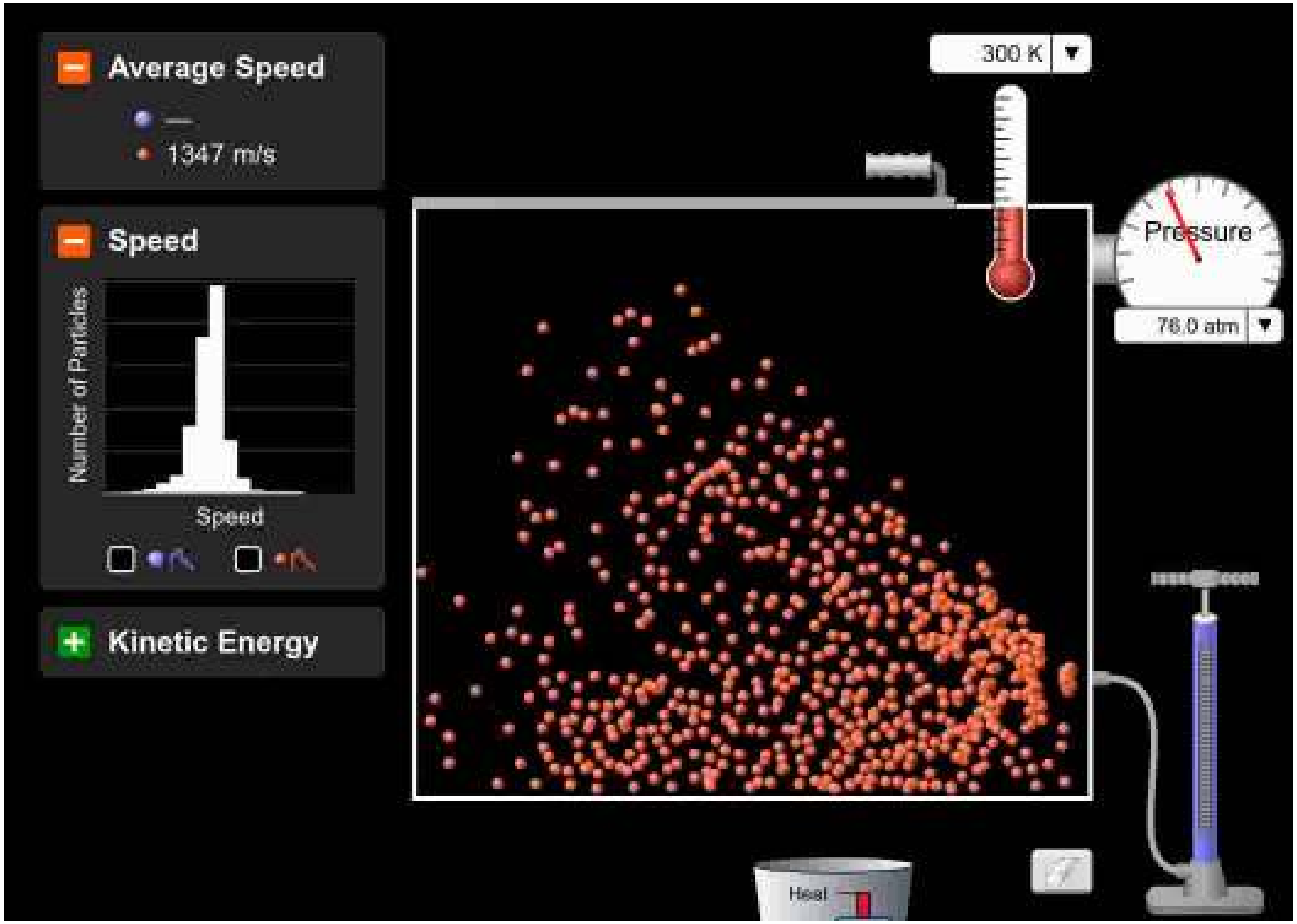
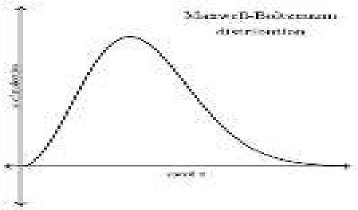
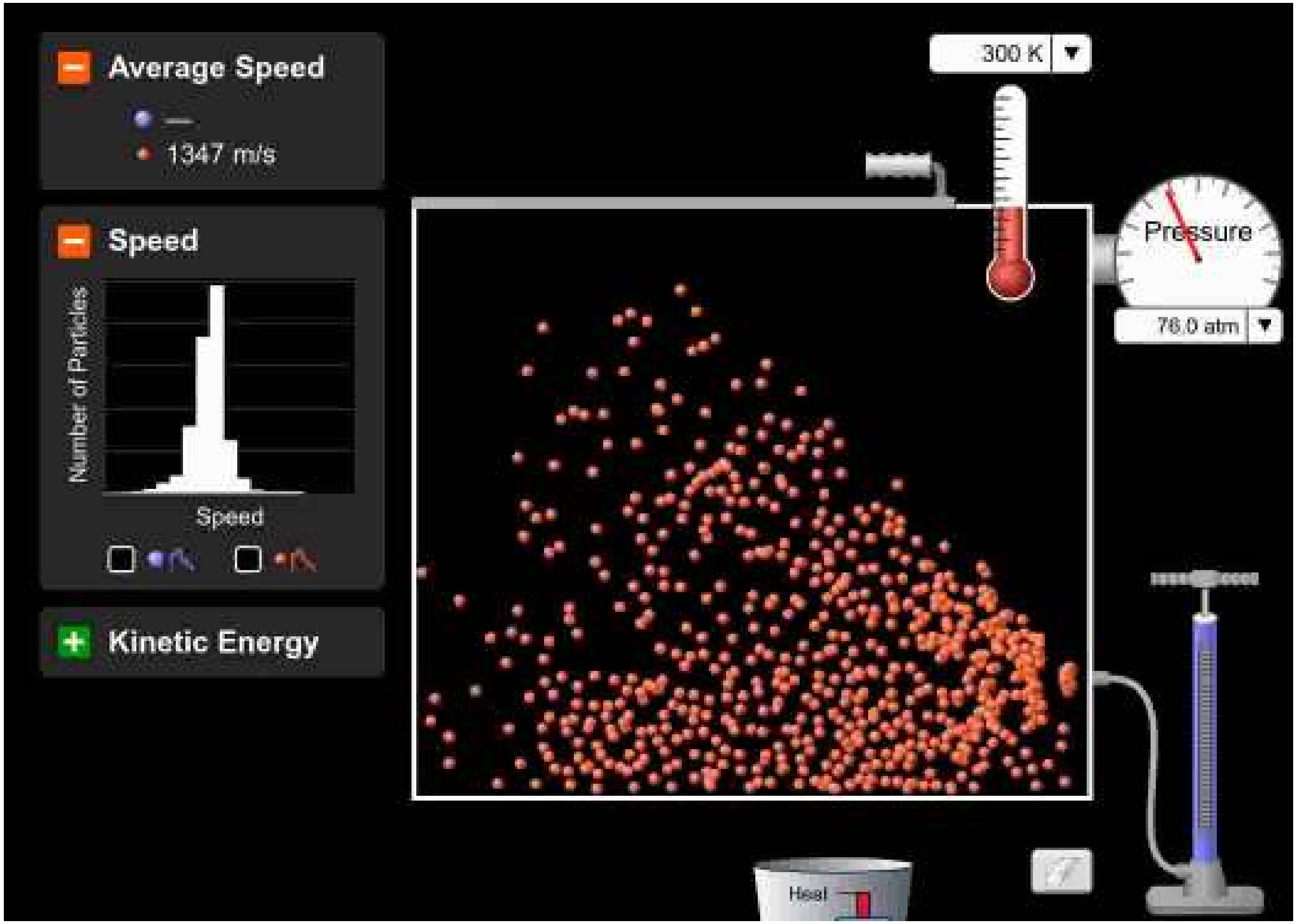


Maxwell-
Boltzmann
distribution
of particle
speeds







Maxwell-Boltzmann distribution of particle speeds

Assuming the particles EQUILIBRATE (or THERMALIZE) we can then describe the system with thermodynamic quantities (Temperature, Pressure, ….)

We don’t need to know the “microscopic” details...

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Conventional Fluids (made of molecules)

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Conventional Fluids (made of molecules)
Quark-Gluon Plasma Fluid

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Quark-Gluon Plasma Fluid
Cold Atom Fluid

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Conventional Fluids (made of molecules)
Quark-Gluon Plasma Fluid
Cold Atom Fluid
Electron Fluid

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Fluid of Gravitating Stars

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Fluid of Gravitating Stars
...

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Ubiquitous $\left\{ \right.$

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Fluid of Gravitating Stars
...

“Let there be Fluid...”

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We *do* need to know what the conservation laws are!

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Conservation of Mass (mass density ρ)
Conservation of Energy (energy density e)

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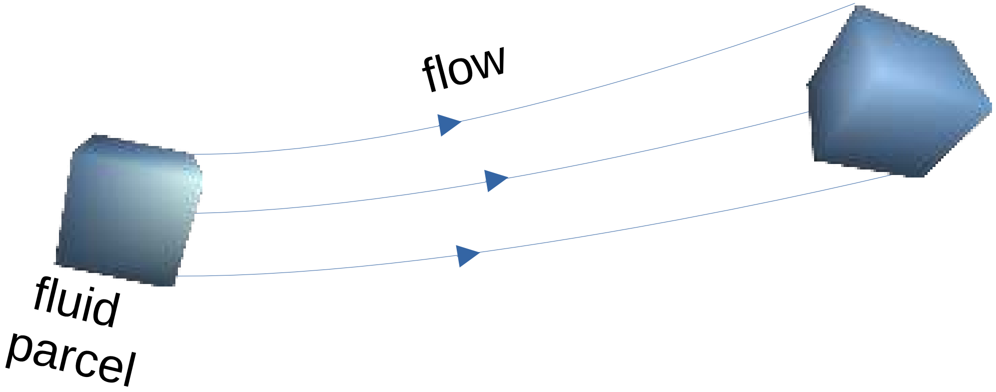

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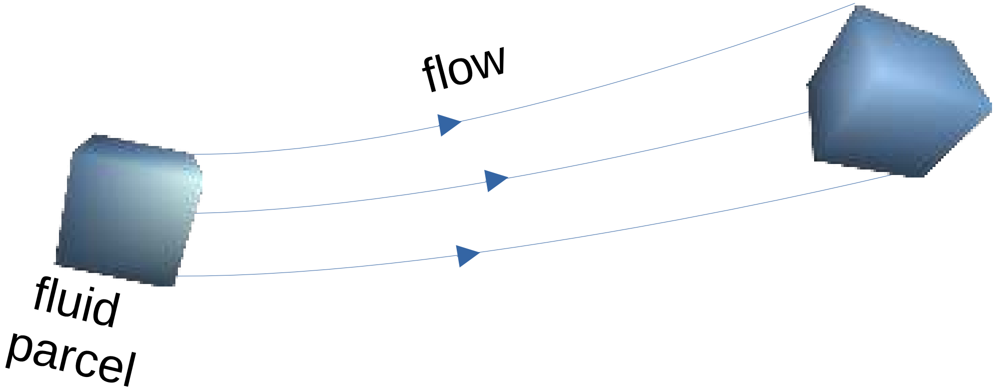
Parcel in
local equilibrium



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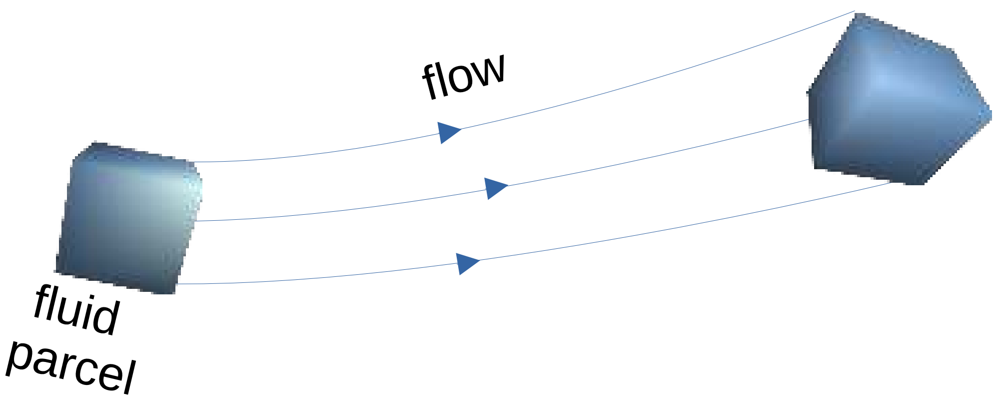

$$\frac{D\rho}{Dt} = 0$$

$$\frac{De}{Dt} = 0$$

$$\frac{D\mathbf{g}}{Dt} = \mathbf{F}$$

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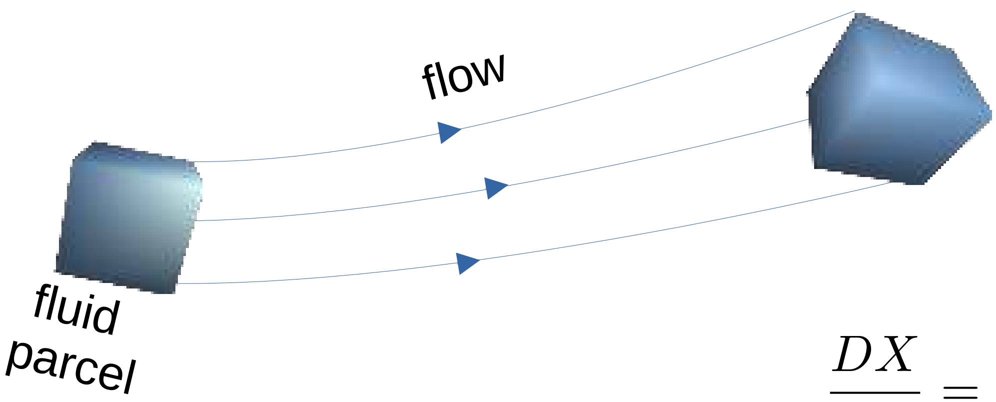

$$\frac{DX}{Dt} = \frac{\partial X}{\partial t} + \mathbf{v} \cdot \nabla X$$

velocity

$$\frac{D\rho}{Dt} = 0$$

$$\frac{De}{Dt} = 0$$

$$\frac{D\mathbf{g}}{Dt} = \mathbf{F}$$

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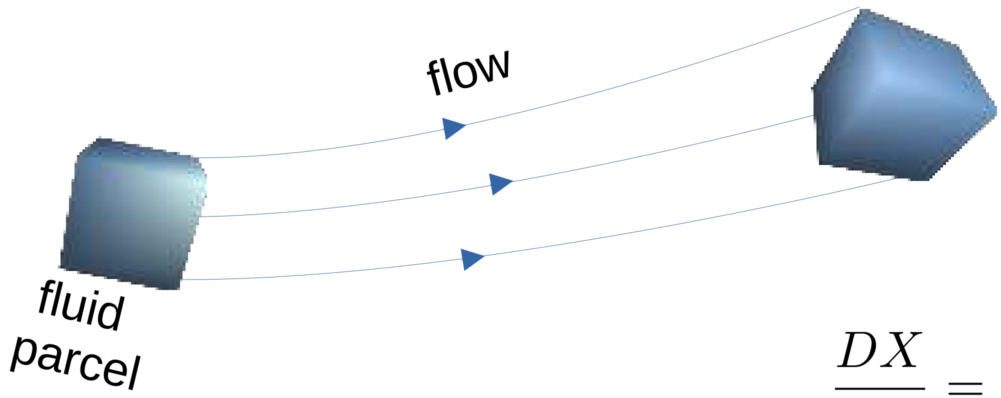

$$\frac{DX}{Dt} = \frac{\partial X}{\partial t} + \mathbf{v} \cdot \nabla X$$

velocity

Incompressible
Euler
Equations

$$\left\{ \begin{aligned} \frac{D\rho}{Dt} &= 0 \\ \frac{De}{Dt} &= 0 \\ \frac{D\mathbf{g}}{Dt} &= \mathbf{F} \end{aligned} \right.$$

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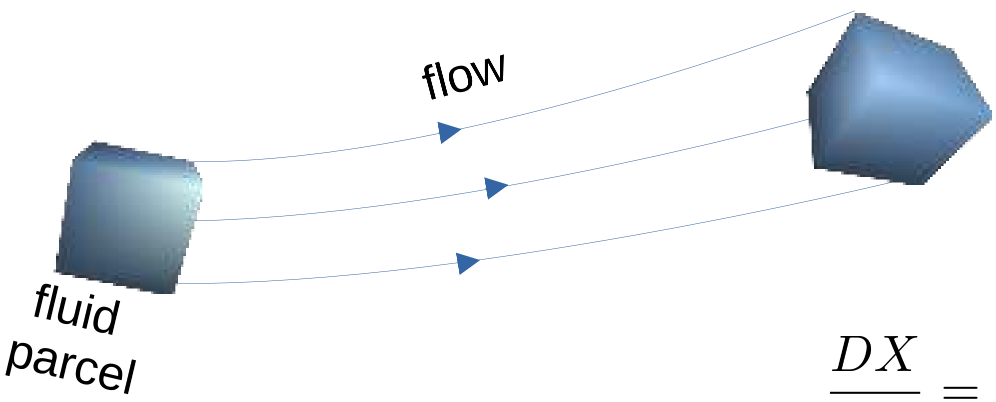

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velocity

Incompressible
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Equations

$$\begin{cases} \dfrac{D\rho}{Dt} = 0 \\[2ex] \dfrac{De}{Dt} = 0 \\[2ex] \dfrac{D\mathbf{g}}{Dt} = \mathbf{F} \end{cases}$$

Navier-Stokes is
from “next order”
corrections

“Hydrodynamics” = dynamics of the conserved quantities

CRUCIAL ASSUMPTION:

everything reduced to just a few conserved (thermodynamic) variables

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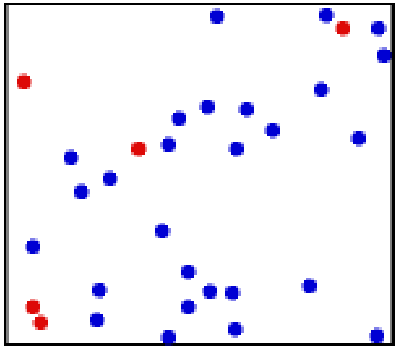

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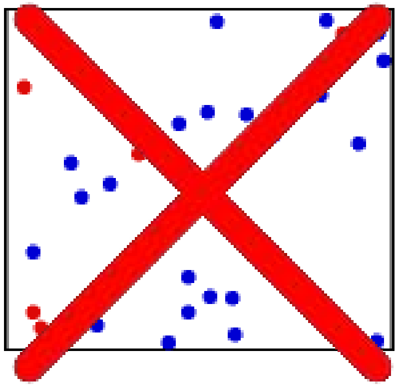

Density
Pressure
Temperature
Mean Velocity

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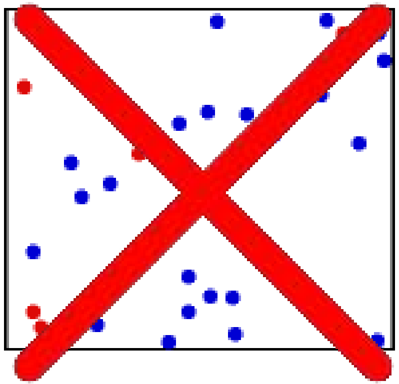

Density
Pressure
Temperature
Mean Velocity

And equation of state relates these together

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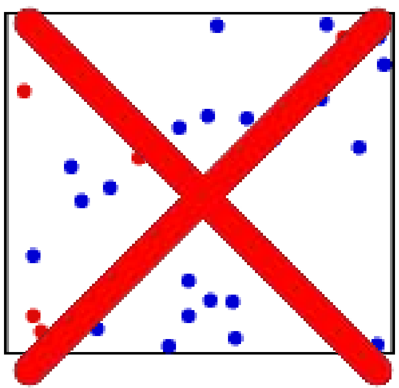

Density
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Mean Velocity

And equation of state relates these together

“Thermodynamics” before “Hydrodynamics”

“Hydrodynamics” = dynamics of the conserved quantities

CRUCIAL ASSUMPTION:
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Bruno
Bertini

Today’s
second
speaker

Some systems have more than
“just a few” conserved variables...

Density
Pressure
Temperature
Mean Velocity

d equation of state relates these together

“Thermodynamics” before “Hydrodynamics”

“Hydrodynamics” = dynamics of the conserved quantities

CRUCIAL ASSUMPTION:
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Bruno
Bertini

Today’s
second
speaker

Some systems have more than
“just a few” conserved variables...

Andrei
Starinets

Today’s
third
speaker

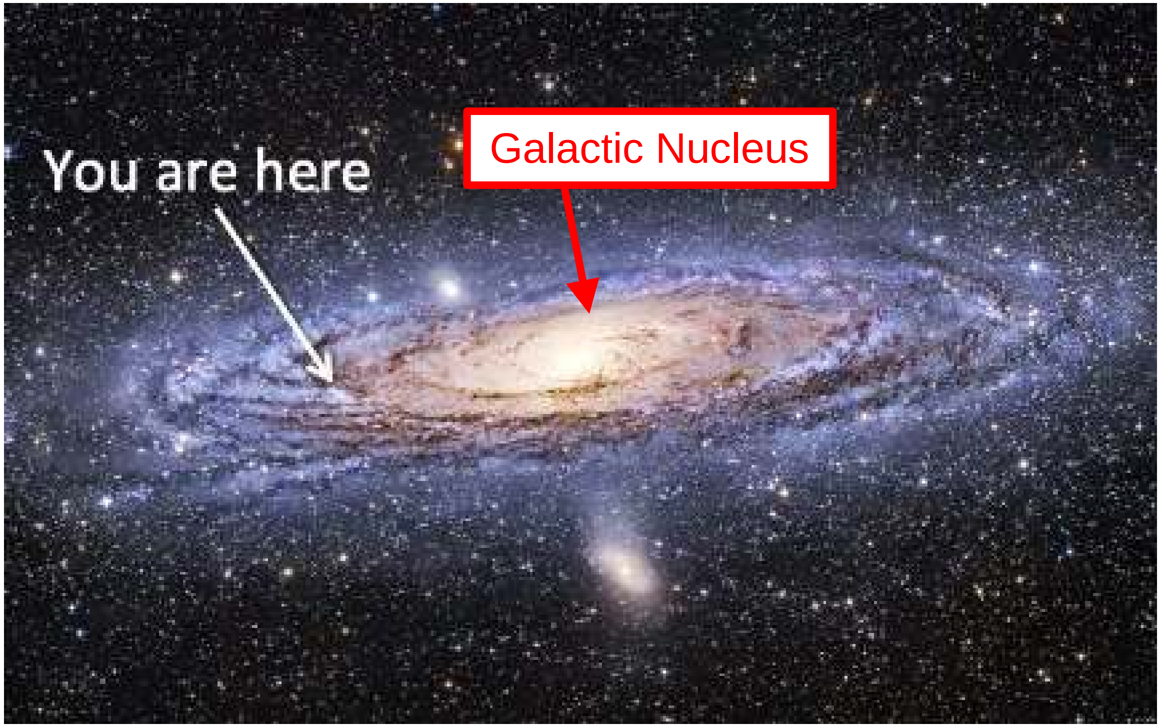
Sometimes Hydrodynamics
before Thermodynamics!

“Thermodynamics” before “Hydrodynamics”

Why Hydrodynamics?

or why not?

Example 1: Flow of Stars



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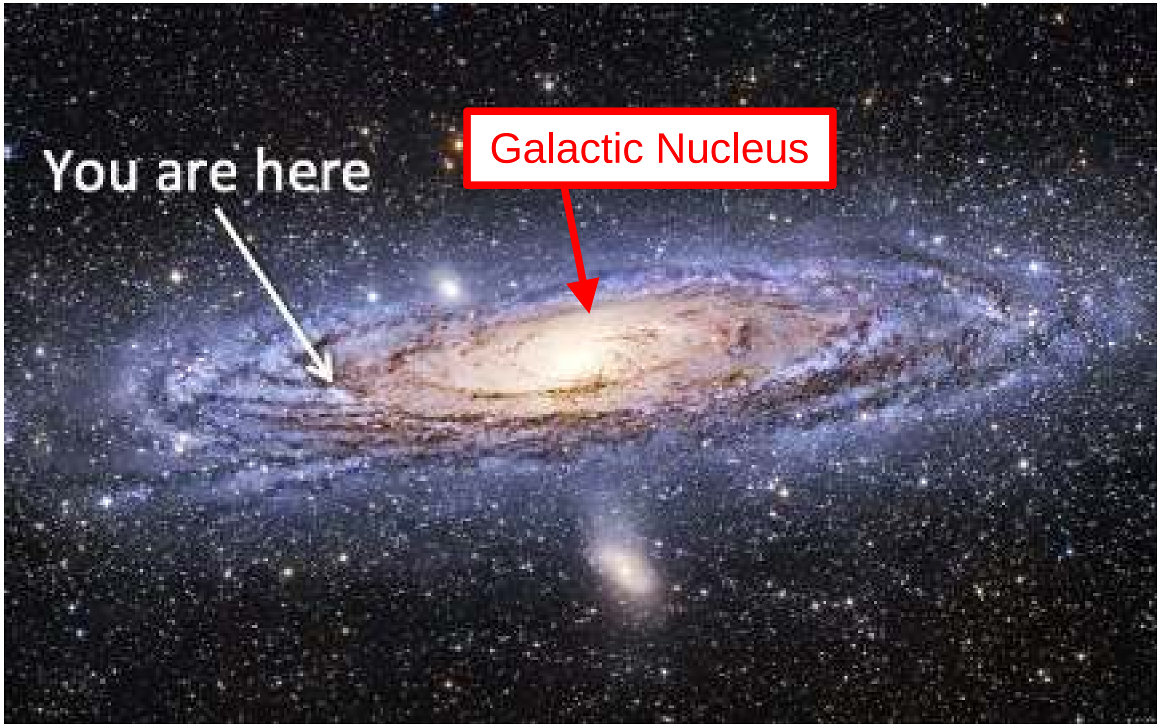

In our part of the galaxy flow of stars is NOT hydrodynamic

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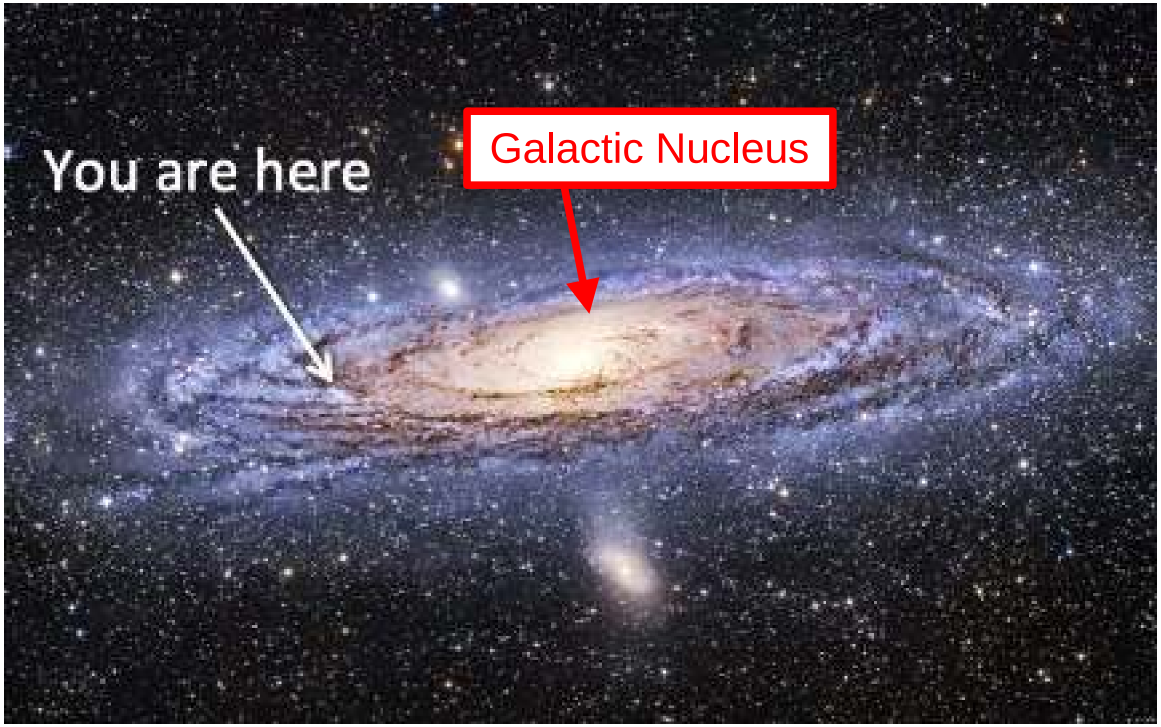
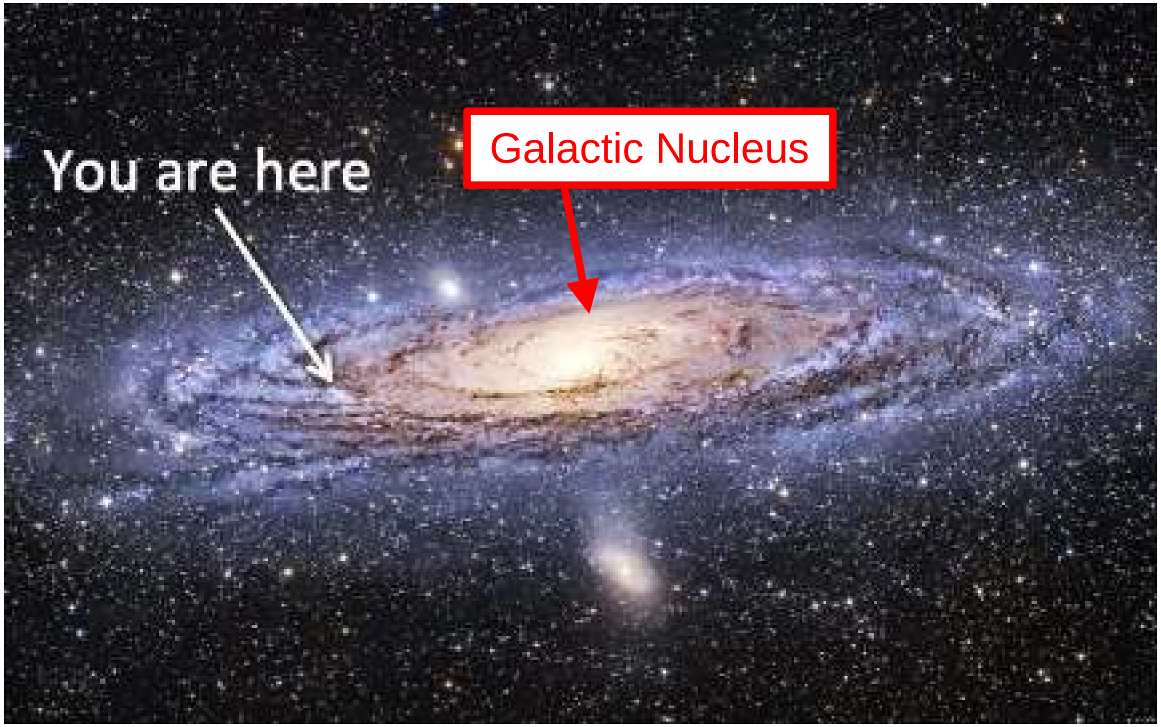


In our part of the galaxy flow of stars is NOT hydrodynamic
(not in thermodynamic equilibrium...)

Example 1: Flow of Stars

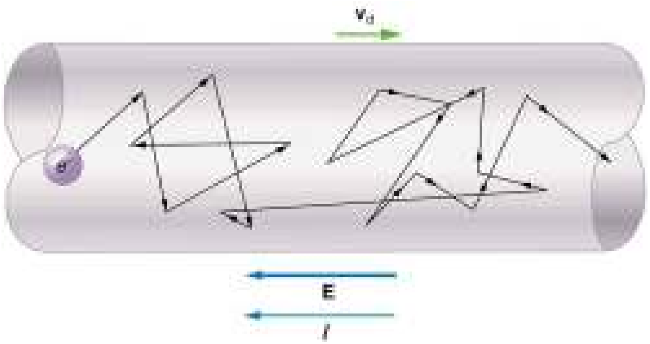


In our part of the galaxy flow of stars is NOT hydrodynamic
(not in thermodynamic equilibrium...)

Near the galactic nucleus it is.

Example 2: Flow of Electrons

1729 electrical flow
(Stephen Gray, Kent)



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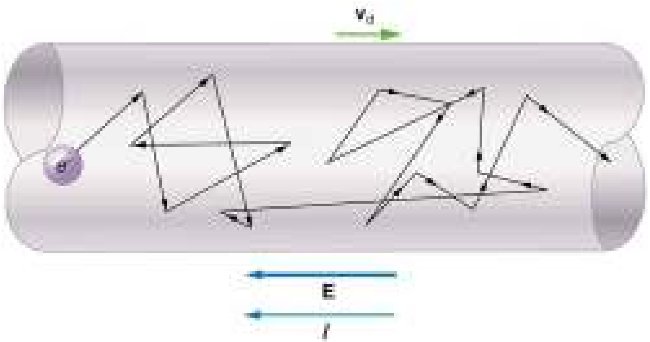

In most materials electrons do not flow hydrodynamically!

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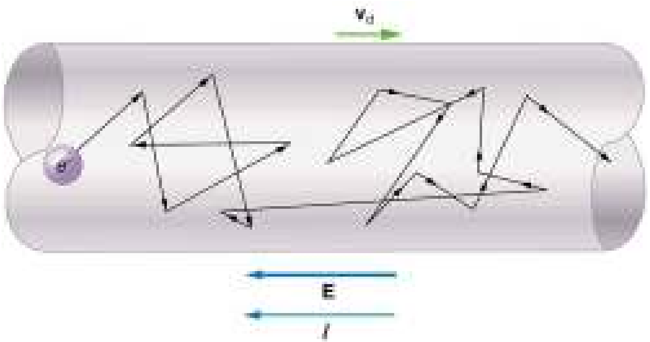

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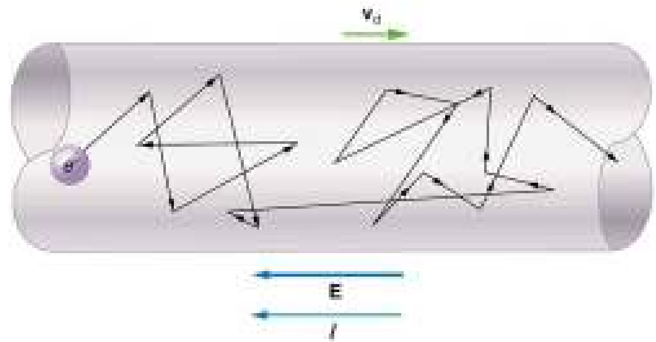

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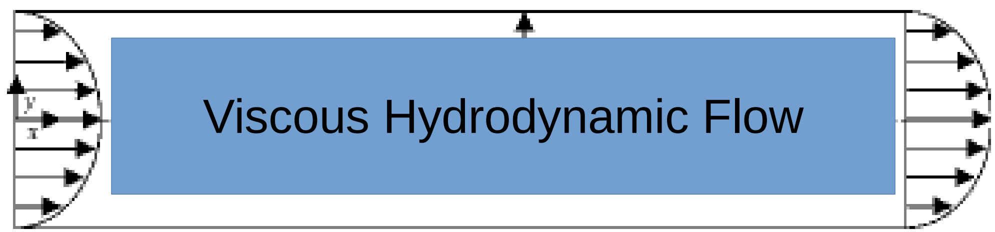
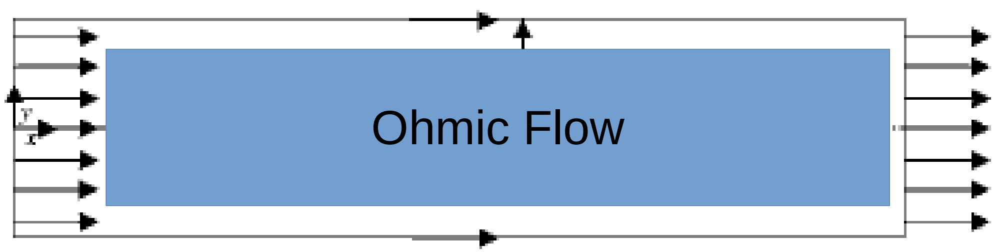
Flow through Pipe



Poiseuille Flow



Uniform Flow

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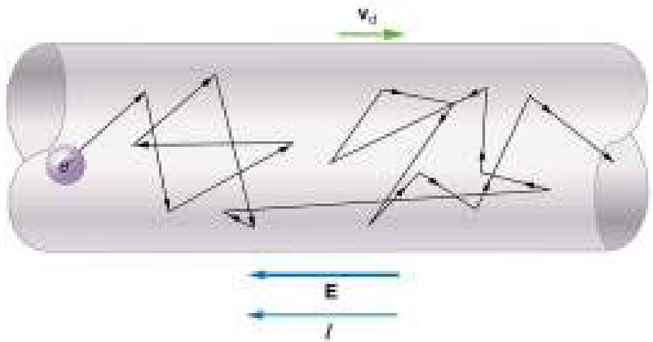

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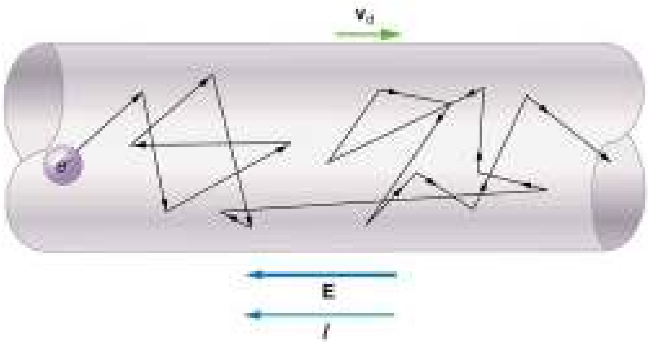

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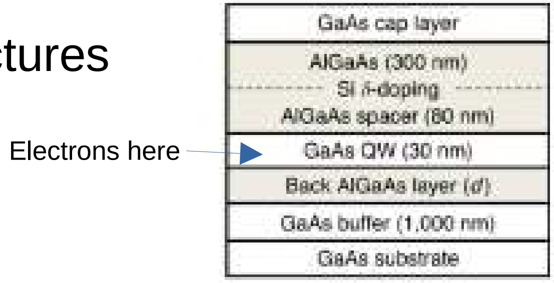

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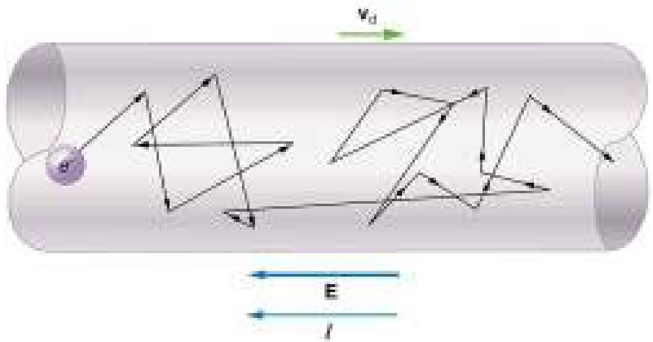

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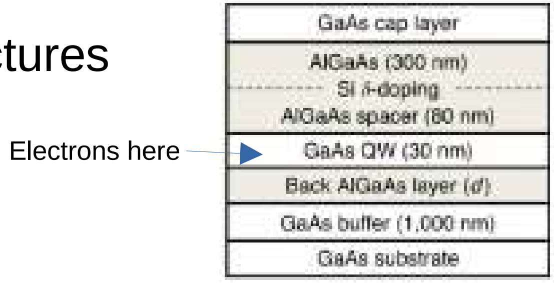

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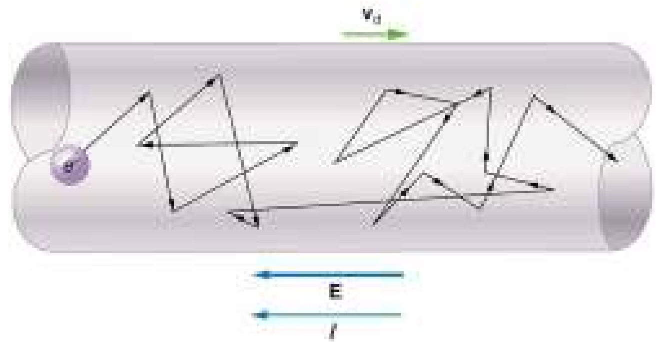

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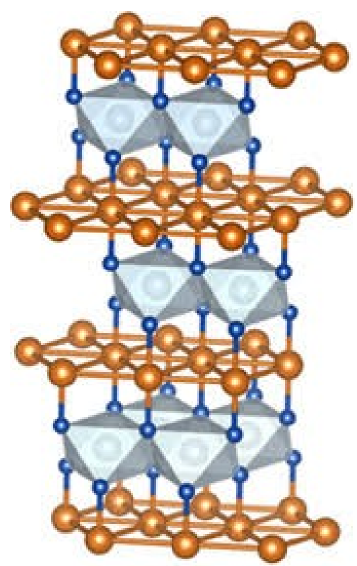

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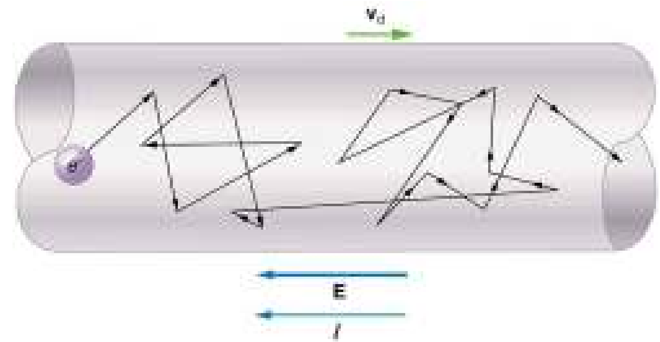

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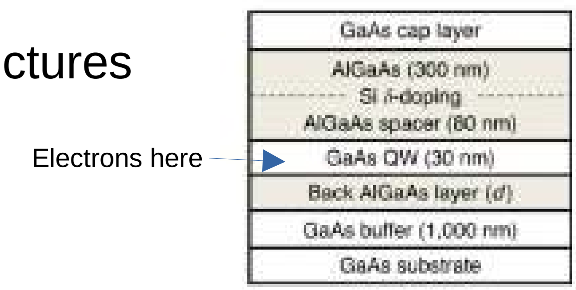

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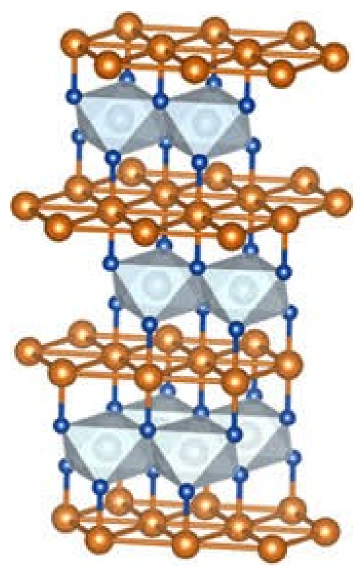

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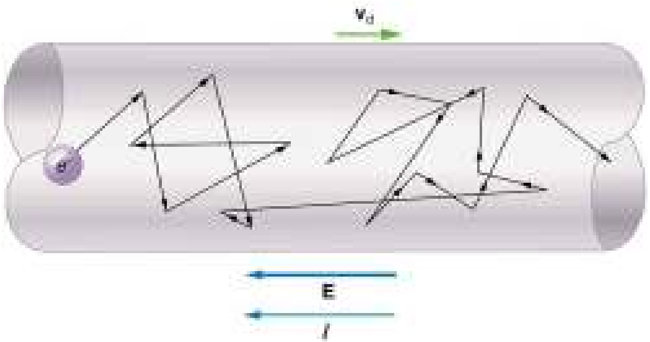

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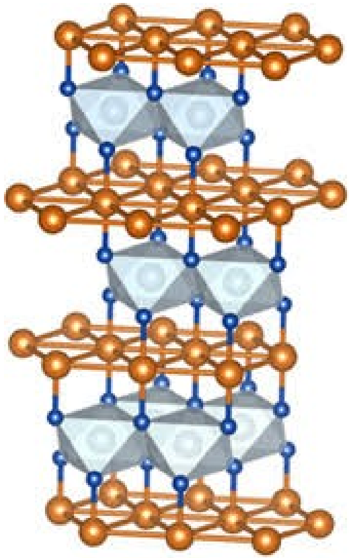

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Hydrodynamics of
electron-phonon fluid

Example 3: Two Oppositely Charged Fluids

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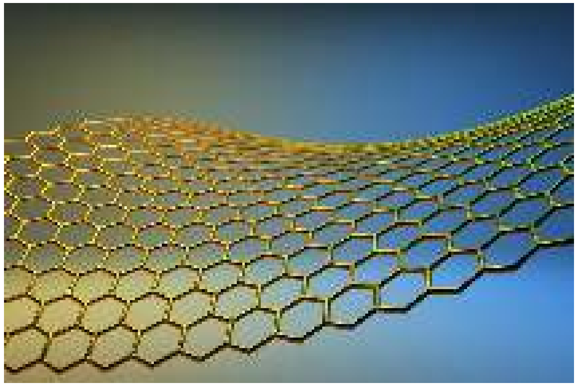
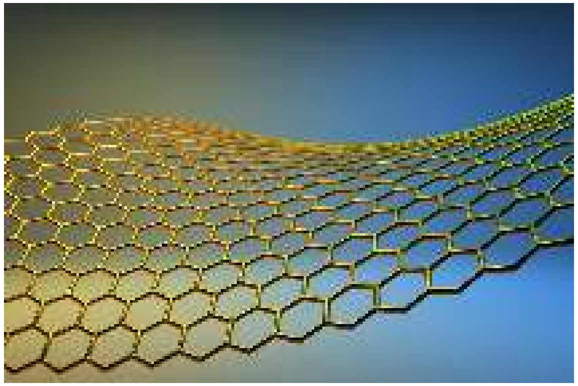
Example 3a: Electron-hole plasma in graphenes



Example 3: Two Oppositely Charged Fluids



Example 3a: Electron-hole plasma in graphenes

Free electrons (-) and Free holes (+)

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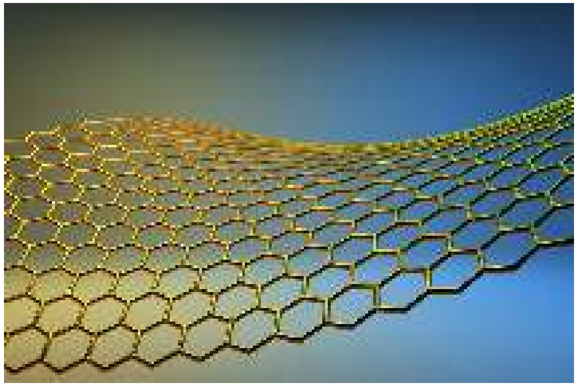

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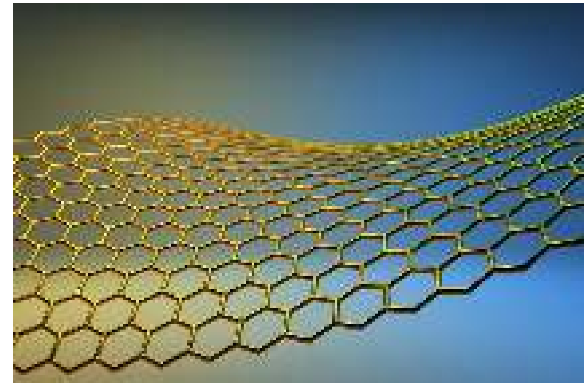

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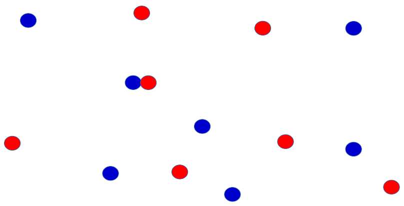

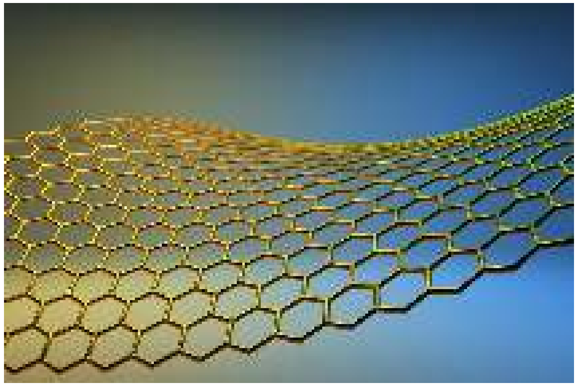

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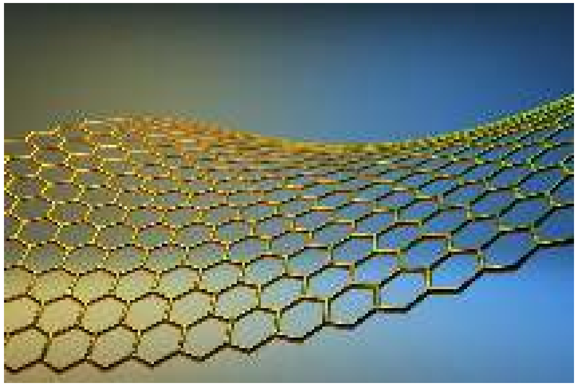

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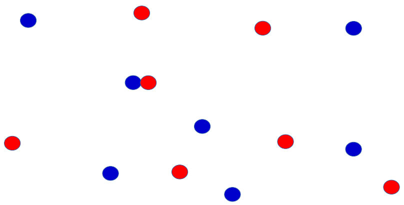

Two (almost) conserved densities = “Two Fluid Hydrodynamics”

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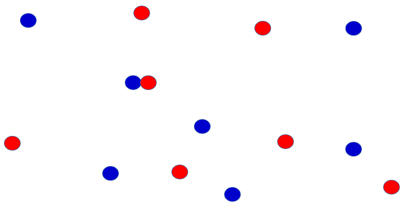

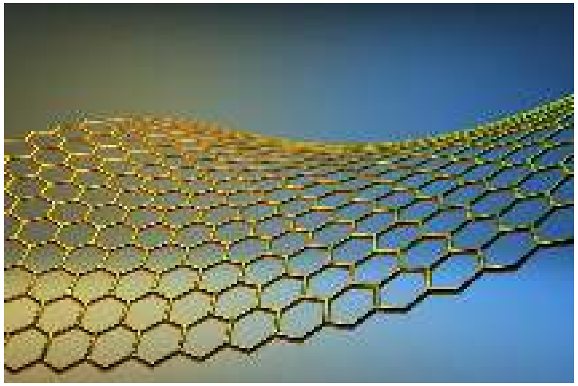

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Momentum and energy can be exchanged between the two

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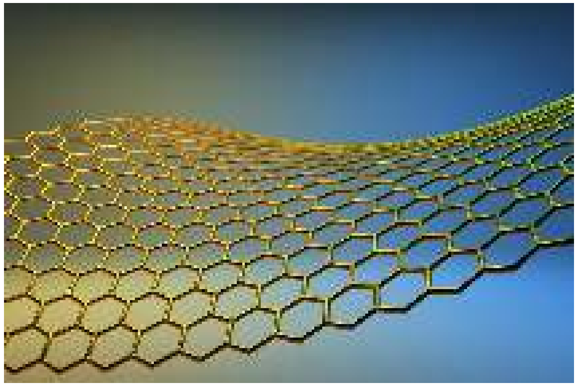

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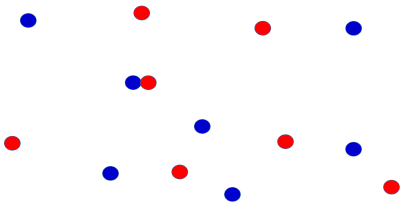

Two (almost) conserved densities = “Two Fluid Hydrodynamics”

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Example 3b: Hydrogen Plasma

Free electrons (-) and Free Protons (+)

Two-fluid (magneto)-hydrodynamics

Plasma flow in the Corona

Images from Solar Dynamics Obervatory

Summary:

Thermodynamics – equilibrated system of many pieces

Hydrodynamics – dynamics of the (almost) conserved quantities

Applies over many many orders of magnitude